

Can we predict the fate of the Universe?

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We re-analyze the question of the use of cosmological observations to infer the present state and future evolution of our patch of the universe. In particular, we discuss under which conditions one might be able to infer that our patch will enter an inflationary stage, as a *prima facie* interpretation of the Type Ia supernovae and CMB data would suggest. We then establish a ‘physical’ criterion for the existence of inflation, to be contrasted with the more ‘mathematical’ one recently proposed by Starkman *et al.* [1].

I. INTRODUCTION

The issue of the present state, future dynamics and final fate of the Universe, or at least our patch of it, has been recently pushed to the front line of research in cosmology. This is mostly due to observations of high redshift type Ia supernovae, performed by two independent groups (the “Supernova Cosmology Project” and the “High-Z Supernova Team”), which allowed accurate measurements of the luminosity-redshift relation out to redshifts up to about $z \sim 1$ [2–4]. It should be kept in mind that these measurements are done on the assumption that these supernovae are standard candles, which is by no means demonstrated and could conceivably be wrong. There are concerns about the evolution of these objects and the possible dimming caused by intergalactic dust [5,6], but we will ignore these for the purposes of this paper, and assume that the quoted results are correct.

The supernovae data, when combined with the ever growing set of CMBR anisotropy observations, strongly suggest an accelerated expansion of the Universe at the present epoch, with cosmological parameters $\Omega_\Lambda \sim 0.7$ and $\Omega_m \sim 0.3$. A further cause of concern here is the model dependence of the CMBR analysis, but we shall again accept the above results for the purpose of this paper.

Taken at face value, these results would seem to show that the universe will necessarily enter an inflationary stage in the near future. However, as pointed out by Starkman, Trodden and Vachaspati [1] this is not necessarily so. We could be living in a small, sub-horizon bubble, for example. And even if we were indeed inflating, it would not be trivial to demonstrate it. In the above work, these authors looked at the crucial question of ‘How far out must we look to infer that the patch of the universe in which we are living is inflating?’ Their analysis is based on previous work by Vachaspati and Trodden [7] which shows that the onset of inflation can in some sense be identified with the comoving contraction of our minimal anti-trapped surface (MAS)¹. They then argue that if one can confirm cosmic acceleration up to a redshift z_{MAS} and detect the contraction of our MAS, then our universe must be inflating. Unfortunately, even if we can do the former (for $\Omega_\Lambda = 0.8$ the required redshift is $z_{MAS} \sim 1.8$), it turns out that there is no way to presently confirm the latter, because the accelerated expansion hasn’t been going for long enough for the MAS to contract. Only if we had $\Omega_\Lambda \geq 0.96$ would we be able to demonstrate inflation today.

As in the proverbial mathematicians joke, the method outlined by Starkman *et al.* provides an answer that is completely accurate but will take a long time to find, and hence is of no immediate use to us. One could, however,

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¹The MAS of each comoving observer is a sphere centered on him/her, on which the velocity of comoving objects is c . For the particular case of an homogeneous universe, the MAS has a physical radius cH^{-1} .

reverse the argument and find the smallest redshift for which, given the currently available observational data, one can say that the universe is *not* inflating. In this paper, however, we will explore a different possibility. Our main aim is to provide what could be called a physicists version of the “mathematical” question of Starkman *et al.* [1]. In other words, we are asking, ‘*If we can’t know for sure the fate of the universe at present, what is our best guess?*’ As we will discuss, we can answer this question, although it will involve making some crucial additional assumptions.

In order to answer the above question we compare the particle and event horizons. We show that for a flat universe with $\Omega_\Lambda \gtrsim 0.14$ the particle horizon is greater than the distance to the event horizon meaning that today we may be able to observe a larger portion of the Universe than that which will ever be able to influence us. We argue that if we find evidence for a constant vacuum density up to a distance from us equal to the event horizon then our Universe will necessarily enter an inflationary phase in the not too distant future, *assuming* that the potential of the scalar field which drives inflation is time-independent and that the content of the observable universe will remain ‘frozen’ in comoving coordinates.

Note that Starkman *et al.* argue that inflation can only take place if the vacuum energy dominates the energy density on a region with physical radius not smaller than that of the MAS at that time. However, they did not assume that the content of the observable universe would remain frozen in comoving coordinates and so they found that the larger is the contribution of a cosmological constant for the total density of the Universe, the larger is the redshift out to which one has to look in order to infer that our portion of the universe is in principle capable of inflating. This result seems paradoxical, until one realizes that the size of the MAS at a given time *does not*, by itself, say anything about inflation. The value of z_{MAS} grows with Ω_Λ simply because the scale factor has grown more.

The plan of this paper is as follows. In the next section we introduce the various lengthscales that are relevant to our discussion, and provide a qualitative discussion of our test for inflation. We also discuss the assumptions involved and compare our ‘physical’ test with the ‘mathematical’ one recently proposed by Starkman *et al.* [1]. In section III we provide a more quantitative analysis of our criterion. We also discuss in more detail our crucial assumption of an energy-momentum distribution which remains frozen in comoving coordinates. Finally, in section IV we summarize our results and discuss some other outstanding issues.

II. A ‘PHYSICAL’ TEST FOR INFLATION

The dynamical equation which describes the evolution of the scale factor a in a Friedmann-Robertson-Walker (FRW) universe containing matter, radiation and a cosmological constant can be written as

$$H^2 = H_0^2(\Omega_{m0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{\Lambda0} + \Omega_{k0}a^{-2}). \quad (1)$$

where $H = \dot{a}/a$ and the density parameters Ω_m , Ω_r and Ω_Λ express respectively the densities in matter, radiation and cosmological constant as fractions of the critical density². Naturally one has $\Omega_k = 1 - \Omega_m - \Omega_r - \Omega_\Lambda$.

The comoving distance d , to a comoving observer at a redshift z is given by

$$d(z) = c \int_{t(z)}^{t_0} \frac{dt'}{a(t')} = cH_0^{-1} \int_0^z \frac{dz'}{[\Omega_{m0}(1+z')^3 + \Omega_{r0}(1+z')^4 + \Omega_{\Lambda0} + \Omega_{k0}(1+z')^2]^{1/2}}, \quad (2)$$

and is related to the ‘radius’ of the local universe which we can in principle observe today. The distance to the *event horizon* can be defined as

$$d_e = c \int_{t_0}^{\infty} \frac{dt'}{a(t')} = cH_0^{-1} \int_1^{\infty} \frac{da}{(\Omega_{m0}a + \Omega_{r0} + \Omega_{\Lambda0}a^4 + \Omega_{k0}a^2)^{1/2}}. \quad (3)$$

and represents the portion of the Universe which will ever be able to influence us³. On the other hand, the *particle horizon*, d_p , is defined by (from eqn. (2))

$$d_p = d_\infty \equiv \lim_{z \rightarrow \infty} d(z), \quad (4)$$

²A dot represents a derivative with respect to the cosmic time t . The subscript ‘0’ means that the quantities are to be evaluated at present epoch, and we have also taken $a_0 = 1$.

³In writing the upper integration limit as infinity we are of course assuming that the universe will keep expanding forever; an analogous formal definition could be given for an universe ending in a ‘big crunch’.

and it represents the maximum distance which we can observe today.

If today the distance to the event horizon is smaller than the particle horizon ($d_p > d_e$) this means that today we are able to observe a larger portion of the Universe than that which will ever be able to influence us. We can do this if we look at a redshift greater than z_* defined by

$$d(z_*) = d_e. \quad (5)$$

In a flat universe solutions to this equation are only possible for $z_* \geq 1$ and for $\Omega_{\Lambda 0} \gtrsim 0.14$. Hence, assuming that the energy-momentum distribution within the patch of the Universe which we are able to see remains unchanged in comoving coordinates, our Universe will necessarily enter an inflationary phase in the future if there is a uniform vacuum density permeating the Universe up to a redshift z_* .

This assumption obviously requires some further discussion. One can certainly think of a universe made up of different ‘domains’, each with its own values of the matter and vacuum energy density. Furthermore, by cleverly choosing the field dynamics, one can always get patches with time-varying vacuum energy densities, or patches where the vacuum energy density is non-zero for only short periods. In all such cases, the domain walls separating these patches can certainly have a very complicated dynamics, and in particular it is always possible that a domain wall will suddenly get inside our horizon sometime between the epoch corresponding to our observations and the present day. On the other hand, it should also be pointed out that a certain amount of fine-tuning would be required to have a bubble coming inside our horizon right after we have last observed it. In these circumstances, the best that can be done is to impose constraints on the characteristics of any bubble wall that could plausibly have entered the patch of the universe we are currently able to observe, given that we have so far seen none. We shall analyse this point in a more quantitative manner in the following section.

We should also point out that if we were to relax the assumption of a co-movingly frozen content of the observable universe, then the equation—analogue to (5)—specifying the redshift out to which one should look in order to be able to predict the future of the Universe would be

$$d(z_+) = d_e(z_+). \quad (6)$$

This equation has no solution, so a stronger test of this kind is not feasible in practice.

We think that the results obtained in this way, even if less robust from a formal point of view, are intuitively more meaningful than those obtained in [1] in the sense that, among other things, in this case the maximum redshift z_* out to which one must observe in order to be able to predict an inflationary phase (subject to the conditions mentioned earlier) decreases as Ω_Λ increases—see Fig. 1. In other words, the larger the present value of the cosmological constant, the easier it should be to notice it.

It is perhaps instructive to compare our test with that of [1] in more detail. Starkman *et al.* require the contraction of the MAS. Now, in order to see the MAS one has to look at a redshift defined by:

$$a(z_{MAS})d(z_{MAS}) - cH^{-1}(z_{MAS}) = 0. \quad (7)$$

This finds the redshift, z_{MAS} , for which the physical distance to a comoving observer at that redshift, evaluated at the corresponding time t_{MAS} , is equal to the Hubble radius at that time. However, if all we wanted was to see a region whose comoving size today was equal to the Hubble radius at that time, one would have instead to look at a redshift z_1 given by:

$$d(z_1) - cH^{-1}(z_1) = 0. \quad (8)$$

We should further note that if the vacuum density already dominates the dynamics of the Universe at the redshift z_1 then eqns. (5) and (8) are equivalent. It should also be kept in mind that during inflation the size of the event horizon is simply equal to cH^{-1} . This is ultimately the reason for the choice of criterium for inflation by Vachaspati and Trodden [7].

To put it in another way, the main difference between our approach and that of ref. [1] lies on the fact that we assume that the energy-momentum content of the observable Universe does not change significantly in comoving coordinates. This allows us to use the equation of state of the local universe observed for a redshift z (looking back at a physical time $t(z)$) to infer the equation of state of the local universe at the present time. In the following section, we shall discuss these points in somewhat more detail.

III. DISCUSSION

Here we go through some specific aspects of our test in more quantitative detail. To begin with, we have solved numerically eqn. (5); the numerical results were obtained for choices of cosmological parameters such

that $\Omega_m + \Omega_\Lambda = 0.7, 1.0, 1.3$, with an additional $\Omega_m = 0.3$ for illustration. We are interested only in a matter-dominated or Λ -dominated epoch of the evolution of the universe, and therefore we have dropped the radiation density parameter Ω_0^r of eqn. (1), in the calculations.

These results are displayed in Fig. 1 as a function of $\Omega_{\Lambda 0}$. The cases with constant total density are shown in solid curves (with the top curve corresponding to the higher value of the density), while the case of a fixed Ω_m , is shown, for comparison purposes, by a dotted curve.

As expected, as the universe becomes more Λ -dominated and/or less matter-dominated, the comoving distance to the event horizon decreases, which is reflected in the decrease of the redshift z_* of a comoving source located at that distance⁴. For the observationally preferred values of $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, the required redshift will be $z_* \sim 1.8$.

For comparison, the redshift, z_{MAS} , corresponding to the MAS, which is the analogous relevant for the criterion of Starkman *et al.* is shown, for the same choices of cosmological parameters, in Fig. 2. Note that in this case, as the universe becomes more Λ -dominated and/or less matter-dominated, the redshift of the MAS will increase. As we already pointed out our test will not be applicable for very low values of the vacuum energy density, and for intermediate values, it requires a higher redshift than z_{MAS} . However, for high values of the cosmological constant and/or low matter contents, the fact that the universe will be expanding much faster makes the redshift of the MAS increase significantly, and even become larger than z_* for some combinations of cosmological parameters. For the same observationally preferred values of Ω_m and Ω_Λ quoted above, the required redshift will be $z_{MAS} \sim 1.6$. A comparison of the values of z_* and z_{MAS} for the spatially flat model is shown in Fig. 3.

We now discuss our assumption about the energy-momentum content of the universe. Let us consider the possibility that different regions of space may have different values for the vacuum energy density, which are separated by domain walls. This means that we are assuming the existence of a scalar field, say ϕ , which within each region sits in one of a number of possible minima of a time-independent potential. It is obvious that if the potential depends on time or if the scalar field did not have time to roll to the minimum of the potential, then it is not possible to predict the fate of the universe without knowing more about the particle physics model which determines its dynamics. For simplicity, we shall assume that we live in a spherical domain with constant vacuum energy density (effectively a cosmological constant) that is surrounded by a much larger region in which the vacuum density has a different value (for the present purposes we can assume it to be zero).

We now consider the possibility that a region with a radius $d(z)$, say centred on a nearby observer, can be inside the domain with non-zero vacuum density at the conformal time $\eta(z)$, but outside that domain at the present time, η_0 . This problem can provide some measure of how good the assumption of a frozen energy-momentum distribution in comoving coordinates is, bearing in mind the possibility that a domain wall may have entered this region at a redshift smaller than z . In order to provide a more quantitative answer to this question, we have performed numerical simulations of domain wall evolution using the PRS [8] algorithm, in which the thickness of the domain walls remains fixed in comoving coordinates for numerical convenience.

We assume that the domain wall has spherical symmetry thus reducing a three-dimensional problem to a one-dimensional one. We perform matter era simulations in a flat universe with no cosmological constant on a one-dimensional 8192 grid. The comoving grid spacing is $\Delta x = c\eta_i$ where $\eta_i = 1$ is the conformal time at the beginning of the simulation. The initial comoving radius of the spherical domain was chosen to be $R = 2048\Delta x$, and the comoving thickness of the domain wall was set to be $10\Delta x$. In these simulations we neglect the gravitational effect induced by the different domains on the dynamics of the universe, and we also do not consider the possibility of an open universe. We must emphasise, however, that both these effects would slow down the defects, thereby helping to justify our assumption of a constant equation of state in comoving coordinates even more. In other words, we are performing the simulation in the least favourable conditions.

We have obtained the following fit for the radius of the domain wall as a function of the conformal time η

$$R(\eta) = R_\infty \left(1 - \left(\frac{c\eta}{\alpha R_\infty} \right)^n \right)^{2/n}, \quad (9)$$

where

$$\alpha = 2.5, \quad n = 2.1 \quad (10)$$

⁴Note that pushing $\Omega_{\Lambda 0}$ down to zero, the value of z_* tends to infinity, since in such universes an event horizon does not exist.

and R_∞ is the initial comoving radius of the domain wall (with $R_\infty \gg c\eta_i$). This fit is accurate to better than 5%, except for the final stages of collapse. In a flat universe with no cosmological constant the comoving distance to a comoving object at a redshift z is given by

$$d(z) = c\eta_0 (1 - 1/\sqrt{1+z}) , \quad (11)$$

whereas the radius of the spherical domain wall can be written as a function of the redshift z , given its initial radius R_∞ , as follows

$$R(z) = R_\infty \left(1 - \left(\frac{c\eta_0}{\alpha R_\infty \sqrt{1+z}} \right)^n \right)^{2/n} . \quad (12)$$

Now, by solving the equation

$$R(z=0) = d(z) \quad (13)$$

we can find the initial comoving radius of our domain (in units of the present conformal time, that is $R_\infty(z)/\eta_0$) which it would be required to have so that its comoving size today is equal to the comoving distance to an object at a redshift z . Finally, we can calculate the radius of this domain at the present time η_0 and at the redshift z (call it $R(z) = R_{\max}$) with the value of $R_\infty(z)/\eta_0$ obtained from the previous equation. In Fig. 4 we plot the value of $R_{\max}/d(z)$ as a function of the redshift, z .

If the radius of our domain at a redshift z was smaller than $d(z)$ the domain wall would be in causal contact with us at the present time and we could in principle detect the gravitational effect both of the domain wall and of the different vacuum density outside our bubble. On the other hand, if the radius of our domain at a redshift z was greater than R_{\max} then it would not have time to enter the sphere of radius $d(z)$ before today. By observing an object at a redshift z we are able to probe a region with a comoving radius equal to $d(z)$.

When the redshift of the cosmological object we are looking at is small, that is ($z \rightarrow 0$), its comoving distance from us, $d(z)$, is much smaller than the comoving horizon, $\eta(z)$, at the time at which the light was emitted. Consequently, a domain wall with a comoving size equal to $d(z)$ at the present time would already have a velocity very close to the speed of light by the redshift z . It is easy to calculate the maximum comoving size, R_{\max} , which our domain would need to have at the redshift z , in order for the domain wall to enter a sphere of comoving radius $d(z)$ centred on a nearby observer sometime between today and redshift z . This is simply given by

$$\frac{R_{\max}}{d(z)} \rightarrow 2 \quad (14)$$

when $z \rightarrow 0$, because $d(z)$ is the distance travelled by light from a redshift z until today.

If we assume that the comoving radius of our bubble at a redshift z is larger than $\eta(z)$, then it will remain frozen in comoving coordinates until its size gets smaller than the horizon. This means that in this case the value of $R_{\max}/d(z)$ is even smaller, approaching

$$\frac{R_{\max}}{d(z)} \rightarrow \frac{\alpha^{-2} \times (\sqrt{1+4\alpha^{-n}} - 1)^{-2/n}}{2^{-2/n}} , \quad (15)$$

when $z \rightarrow \infty$. For a spherical domain we have

$$\frac{R_{\max}}{d_\infty} \approx 1.12 . \quad (16)$$

We thus see that for the purposes of predicting the fate of the universe it may be a plausible assumption to assume a fixed content in comoving coordinates. The above discussion also suggests, in particular, that one may find *a posteriori* that it is indeed a reasonable assumption if we can observe the dynamical effects of a uniform vacuum density up to a redshift $z \geq 1$.

IV. CONCLUSIONS

We have provided a simple analysis of the use of cosmological observations to infer the state and fate of our patch of the universe. In particular, in the same spirit of Starkman *et al.* [1], we have discussed possible criteria for inferring the present or future existence of an inflationary epoch in our patch of the universe.

We have presented a ‘physical’ criterion for the existence of inflation, and contrasted it with the ‘mathematical’ one that has been introduced in [1]. Ours has the advantage of being able to provide (in principle) a definite answer at the present epoch, but the disadvantage of ultimately relying on assumptions on the content of the local universe and on field dynamics. We consider our assumptions to be plausible, but we can certainly conceive of (arguably contrived or fine-tuned) mechanisms that would be capable of violating it.

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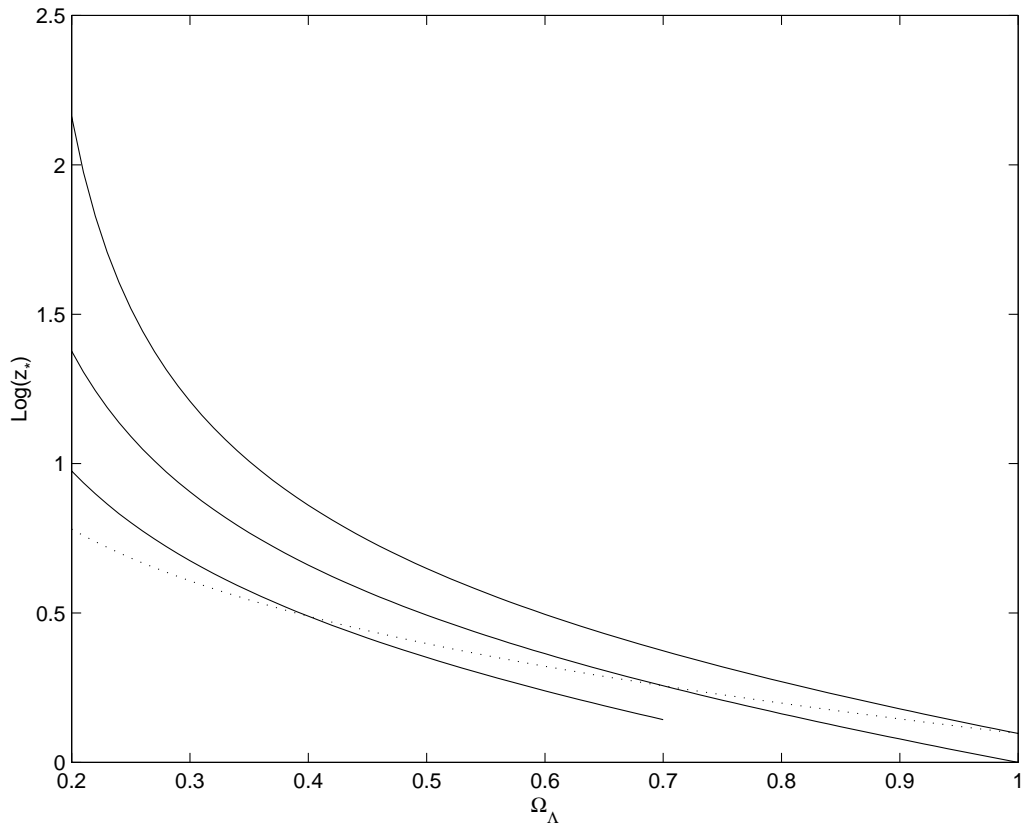


FIG. 1. The solution of eqn. (5) for cosmologies with $\Omega_m + \Omega_\Lambda = 0.7, 1.0, 1.3$ (solid curves, bottom to top) and $\Omega_m = 0.3$ for illustration (dotted curve). Observing a uniform vacuum energy density up to a redshift z_* will imply that our universe will enter an inflationary phase in the future, subject to the conditions specified in the text.

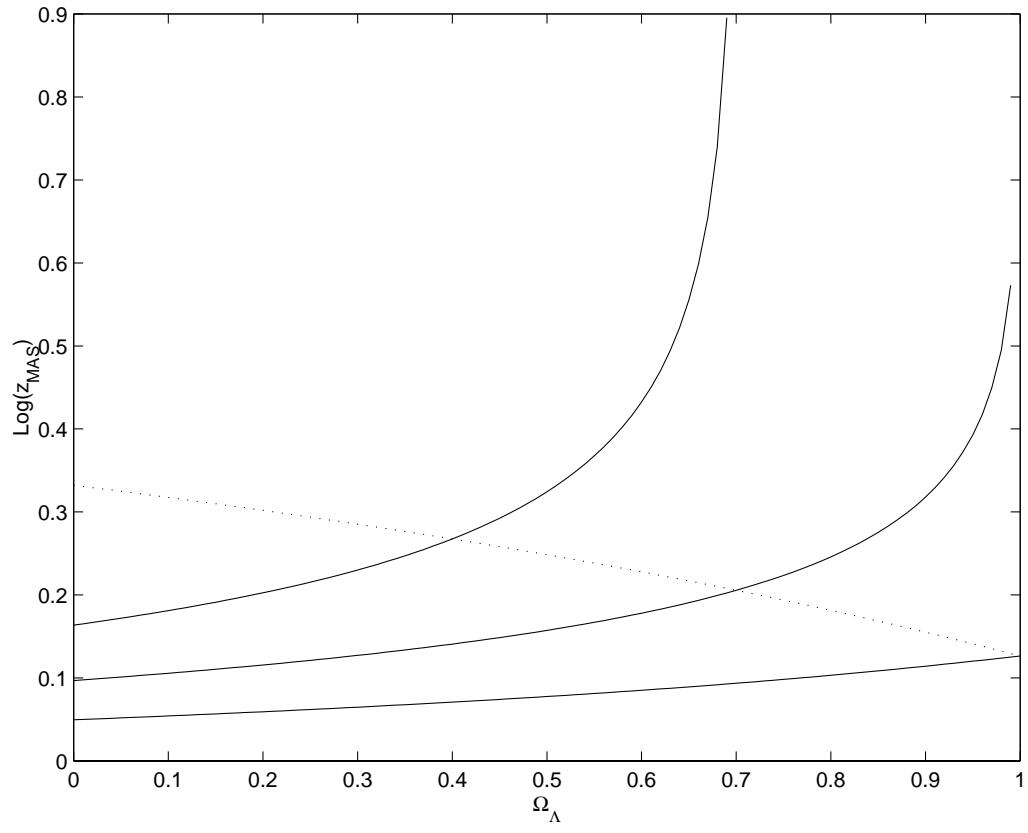


FIG. 2. The redshift of the MAS, relevant for the inflationary criterion of Starkman *et al.*, for the same cosmological models as in Fig. 1. Note that the solid curves for $\Omega_m + \Omega_{\Lambda} = 0.7, 1.0, 1.3$ now appear in the graph from top to bottom.

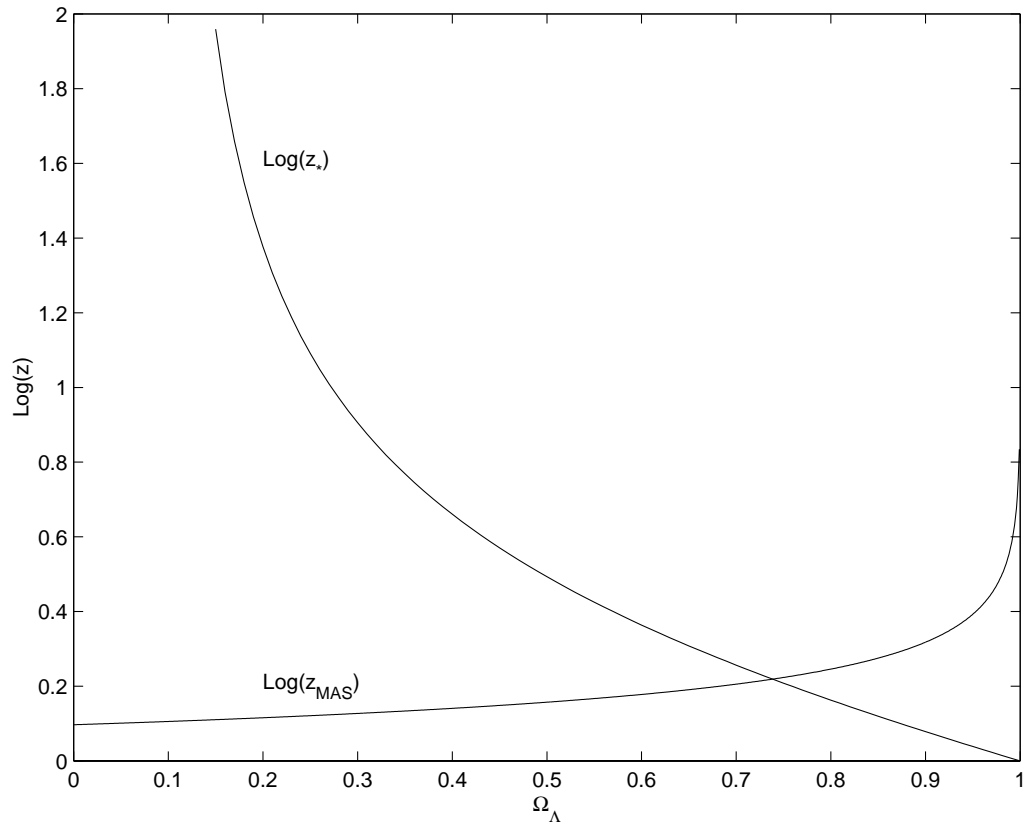


FIG. 3. Comparing the values of the critical redshifts z_* and z_{MAS} , as a function of the vacuum energy density, for the spatially flat models ($\Omega_m + \Omega_\Lambda = 1.0$).

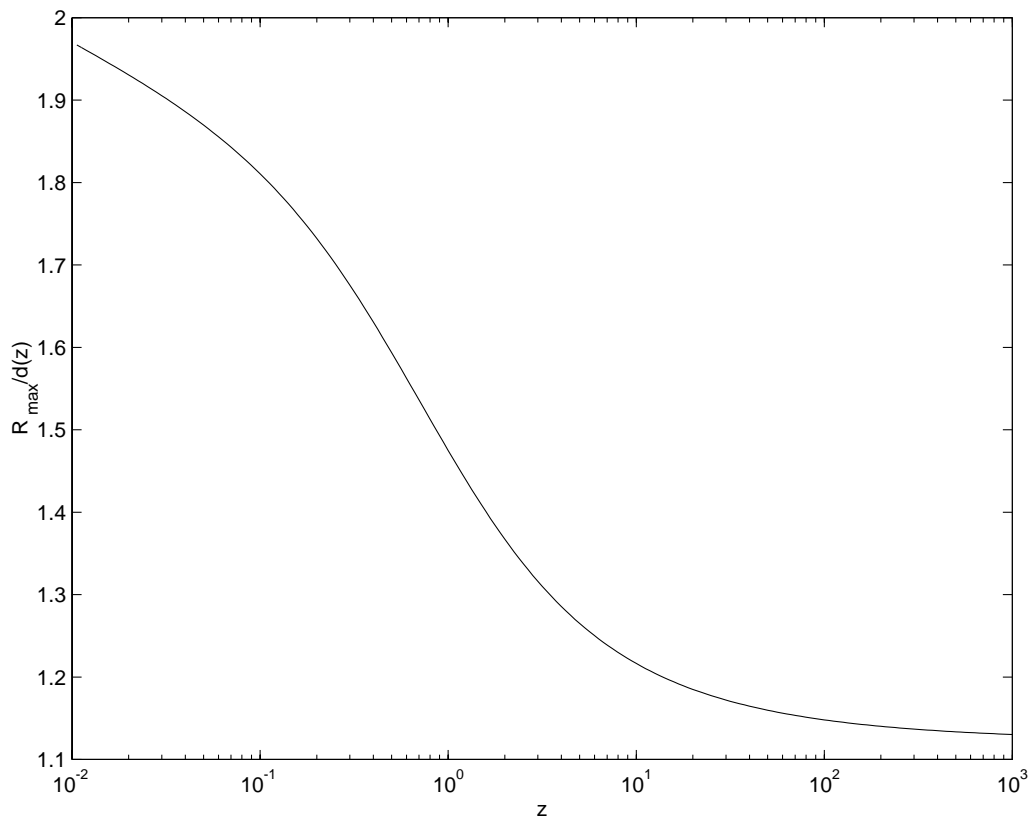


FIG. 4. The comoving radius of a domain wall at redshift z whose present comoving size equals the comoving distance to an object at redshift z —denoted $d(z)$, see (11)—in units of $d(z)$, as a function of redshift.